EXERCISE 7.1

Q1. In quadrilateral ACBD, AC = AD and AB bisects ∠A (see Fig. 7.16). Show that Δ ABC ≅ Δ ABD.

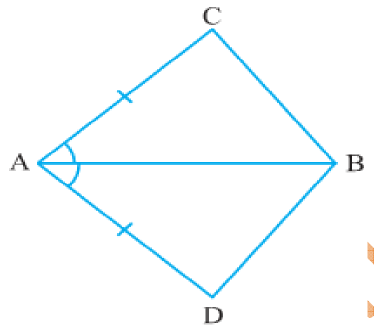


Fig. 7.16

Solution:

Given: AC = AD and AB bisects ∠A

To prove: Δ ABC ≅ Δ ABD.

Proof: In Δ ABC and Δ ABD.

AC = AD [given]

∠CAB = ∠BAD [AB bisect ∠A]

AB = AB [Common]

By SAS Congruence Criterion Rule

Δ ABC ≅ Δ ABD

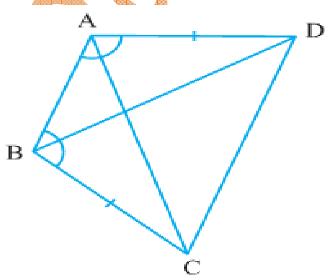
BC = BD [CPCT]

Q2. ABCD is a quadrilateral in which AD = BC and ∠ DAB = ∠ CBA (see Fig. 7.17). Prove that

(i) Δ ABD ≅ Δ BAC

(ii) BD = AC

(iii) ∠ ABD = ∠ BAC



Solution:

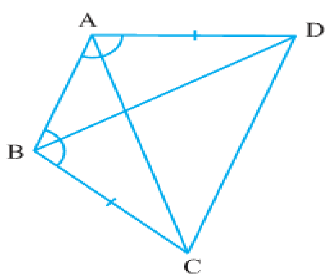
Given: ABCD is a quadrilateral in which AD = BC and ∠ DAB = ∠ CBA

To prove:

(i) Δ ABD ≅ Δ BAC

(ii) BD = AC

(iii) ∠ ABD = ∠ BAC



Proof: In Δ ABD and Δ BAC

AD = BC [given]

∠ DAB = ∠ CBA [given]

AB = AB [Common]

By SAS Congruence Criterion Rule

Δ ABD $\cong$ Δ BAC

BD = AC [CPCT]

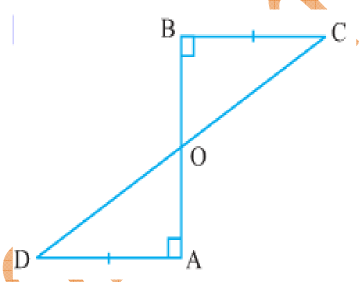
$\angle$ ABD = $\angle$ BAC [CPCT]

Q3: AD and BC are equal perpendiculars to a line segment AB (see the given figure). Show that CD bisects AB.

Solution:

Given: AD and BC are equal perpendiculars to a line segment AB.

To prove: CD bisects AB.



Proof: In ΔBOC and ΔAOD

$\angle$ BOC = $\angle$AOD (Vertically opposite angles)

$\angle$CBO = $\angle$DAO (Each 90º)

BC = AD (Given)

By AAS Congruence Criterion Rule

ΔBOC $\cong$ ΔAOD

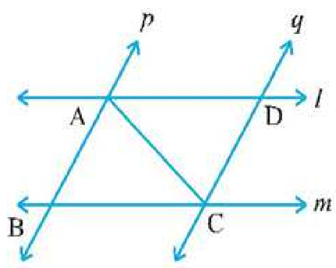
BO = AO (By CPCT)

Hence, CD bisects AB.

Q4. l and m are two parallel lines intersected by another pair of parallel lines p and q (See the given figure). Show that ΔABC $\cong$ ΔCDA

Solution:

Given: l and m are two parallel lines intersected by another pair of parallel lines p and q.



To prove: ΔABC $\cong$ ΔCDA

Proof: In ΔABC and ΔCDA,

$\angle$ BAC = $\angle$DCA (Alternate interior angles, as p || q)

AC = CA (Common)

$\angle$ BCA = $\angle$DAC (Alternate interior angles, as l || m)

By AAS Congruence Criterion Rule

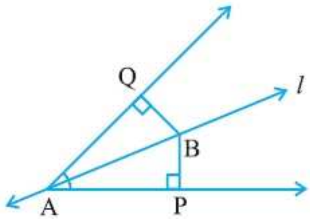
ΔABC $\cong$ ΔCDA

Q5. Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of a (see the given figure).

Show that:

(i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.



Solution:

Given: Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of a.

To prove: (i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.

Proof: In ΔAPB and ΔAQB,

∠ APB = ∠AQB (Each 90º)

∠ PAB = ∠QAB (l is the angle bisector of A)

AB = AB (Common)

By AAS Congruence Criterion Rule
ΔAPB ≅ ΔAQB

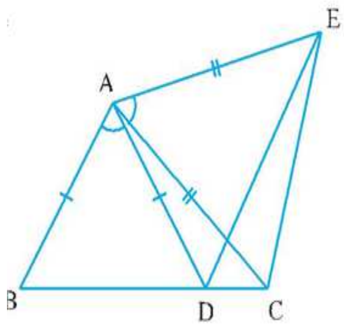
BP =BQ **[By CPCT]**

it can be said that B is equidistant from the A.

Q6. In the given figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.

Solution:
Given: AC = AE, AB = AD and ∠BAD = ∠EAC.

To prove: BC = DE.



Proof: ∠BAD = ∠EAC (Given)

Adding ∠DAC in both the sides,
∠BAD + ∠DAC = ∠EAC + ∠DAC

BAC = DAE

In ΔBAC and ΔDAE

AC = AE (Given)

AB = AD (Given)

∠BAC = ∠DAE (proved above)

By SAS Congruence Criterion Rule

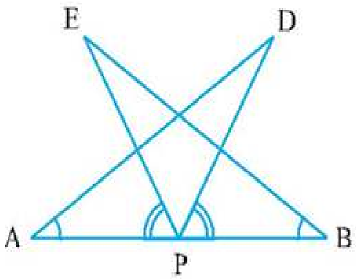
ΔBAC ≅ ΔDAE

BC = DE (CPCT)

Q 7: AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD =∠ ABE and ∠EPA = ∠DPB (See the given figure). Show that :
(i) ΔDAP ≅ ΔEBP
(ii) AD = BE

Solution:

Given: AB is a line segment and P is its mid-point. D and E are points on the same side Of AB such that ∠BAD =∠ ABE and ∠EPA = ∠DPB.



To prove:
(i) ΔDAP ≅ ΔEBP
(ii) AD = BE

Proof: ∠EPA = ∠DPB …. (Given)

Adding ∠DPE in both the sides

∠EPA + ∠DPE = ∠DPB + ∠DPE

∠ DPA = ∠EPB

In Δ DPA and Δ EPB

∠BAD = ∠ ABE (Given)

∠EPA = ∠DPB (Given)

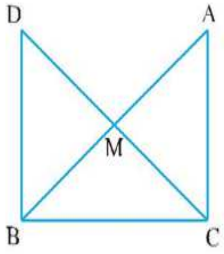
AP = BP (P is the midpoint of AB)

By AAS Congruence Criterion Rule

ΔDAP ≅ ΔEBP

AD = BE (By CPCT)

Q8. In right triangle ABC, right angled at C, M is The mid-point of hypotenuse AB. C is join To M and produced to a point D such that DM = CM. Point D is joined to point D is joined to point B. Show that:



(i) ΔAMC ≅ ΔBMD

(ii) ∠ DBC is a right angle

(iii) DBC ≅ ABC

(iv) CM = $\frac{1}{2}$ AB

Solution:

Given: In right triangle ABC, right angled at C, M The mid-point of hypotenuse AB. C is join To M and produced to a point D such that DM = CM. Point D is joined to point D is Joined to point B.

To prove:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle

(iii) DBC ≅ ABC

(iv) CM = $\frac{1}{2}$ AB

Proof: In ΔAMC and ΔBMD

AM = BM (M is the mid-point of AB)

∠AMC = ∠BMD (Vertically opposite angles)

CM = DM (Given)

By SAS Congruence Criterion Rule

ΔAMC ≅ ΔBMD
AC = BD (By CPCT)
∠ACM = ∠BDM (By CPCT)

(ii) ∠ACM = ∠BDM (A.I.A)
It can be said that DB || AC

∠DBC + ∠ACB = 180º (Co-interior angles)

∠DBC + 90º = 180º

∠DBC = 90º

Therefore, ∠DBC is a right angle.

(iii) In ΔDBC and ΔACB

DB = AC (Already proved)

∠DBC = ∠ACB (Each 90^0)

BC = CB (Common)

By SAS Congruence Criterion Rule

$\Delta DBC \cong \Delta ACB$

(iv) AB = DC (By CPCT)

Therefore, AB = 2 CM

$CM = \frac{1}{2} AB$ **Proved**